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Design and simulation of X-rudder control allocator for Submarine



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Abstract: [Objectives] To solve the steering control problem of X-rudder submarines under different available rudder surfaces conditions, the rudder control distributor of an X-rudder submarine is designed. [Methods] A cascade control structure including a main controller and control allocator is adopted in the design of the steering control system. The main controller adopts an existing mature algorithm, and a constrained control allocation algorithm is designed by converting the rudder angle control allocation problem with the rudder angle and rudder speed limit to solve the weighted least square (WLS) problem. The control allocation algorithm is verified through a simulation of turning at certain depths and diving in three operation cases (i.e. quadruple rudder, triple rudder and double rudder). [Results] Under the given rudder conditions, with the decrease in the steering surface, the difference in the submarine's diving performance is not significant, turning performance is reduced slightly and heeling and rudder angle oscillation tends to be serious. [Conclusions] The simulation results show that the control allocation algorithm is reasonable and suitable for different rudder configurations.

Key words: submarine; X-rudder; control allocation; weighted least squares (WLS)

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0 Introduction

At present, the submarines are mainly equipped with two kinds of control device arrangements, namely cruciform (or cross) and X rudder configuration. Compared with rudders arranged in cross configurations (hereinafter referred to as "C-rudders"), those in X configuration (hereinafter referred to as "X-rudders") provide improved maneuverability and safety. Thus, X-rudders have been widely used in designing stern appendages of submarines, e.g., that used in the conventional submarines like Swedish Gotland-class, Australian Collins-class, German class 212A, and Japanese Soryu-class, as well as that used in the nuclear submarines, such as French Baracuda-class attack submarine (SSN), and the Columbia-class strategic submarine (SSBN) under construction in the US. However, X-rudders are also

faced with major problems like indirect manual steering. X-rudder steering-control systems mainly employ computer-based automatic control. Therefore, it is crucial to design steering control algorithms with high performance and reliability.

In terms of motion modeling of X-rudder submarines, Hu^[1] established a mathematical model for rudder-angle transition between X-rudders and C-rudders and verified the device designed for equivalent rudder-angle transition by simulation. In terms of X-rudder control algorithms, Zeng^[2] studied X-rudder PID control of an AUV under a diagonally coordinated operation mode and carried out both lake and sea trial. However, the master controller adopted a PID-control algorithm, with an allocation mode of diagonal coordinated operation, and allocation problems under triple-rudder and double-rudder conditions were not considered. Zhang et

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al.^[3] introduced a rudder-angle allocator into the traditional control structure of C-rudder AUV and studied fault-tolerant control allocation of X-rudders based on a reconfiguration method. In addition, some scholars conducted in-depth study on algorithms of multiple surface-control allocation based on a pseudo-inverse technique^[4-5]. The advantage of the pseudo-inverse method is that calculation is relatively simple without considering the constraints of steering mechanisms. However, in practical application, it is difficult to use a pseudo-inverse technique to solve constrained control-allocation problems.

In view of X-rudder control-allocation problems of submarines, this paper adopted a steering system with a "master controller-control allocator" cascade control structure, as well as a master controller with a mature algorithm. In addition, under the consideration that steering gear are constrained by limitation of rudder amplitudes and rates, the paper designed an X-rudder control allocator of submarines through weighted least squares (WLS) to realize multiple surface-control allocation. Finally, it verified steering control effects under different available rudder surfaces through simulation. The research in this paper can provide a certain reference for practical engineering application.

1 Modeling of steering motion of X-rudder submarine

All of the coordinate systems, terminologies, and nomenclatures used in this paper originate from the system recommended by ITTC and SNAME^[6-7]. Fig. 1 shows the corresponding fixed coordinate system $E-\xi\eta\zeta$ and motion coordinate system $O-xyz$. In the figure, components of the force acting on the submarine in the motion coordinate system are as follows: longitudinal force X , transverse force Y , vertical force Z , heeling moment K , trimming moment M , and yawing moment N .

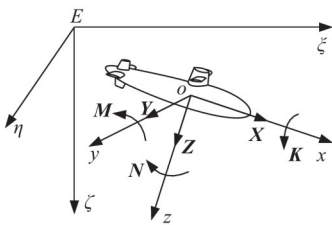


Fig. 1 Fixed coordinate system and motion coordinate system

1.1 X-rudder layout and rudder-force analysis

X-rudder steering surfaces are generally com-

posed of rudder surfaces and stabilizing wings. The stabilizing wings and the hull are fixedly connected orthogonally in an X configuration, with an angle of 45° between centerlines of rudder axes and the center-line plane of the submarine. Rudder surfaces can rotate within a certain range. Total rudder force F_x is a spatial force, expressed as $F_x = [X_x, Y_x, Z_x, K_x, M_x, N_x]$. Any rudder-surface deflection will result in such motion as diving, turning, and heeling of the submarine. As shown in Fig. 2, for the component F_i of F_x in the cross-section $y'O'z'$ of the hull at the center O' of a rudder axle, its relationship to transverse force Y_{xi} and vertical force Z_{xi} is as follows:

$$\begin{cases} Y_{xi} = F_i \cdot \cos 45^\circ \\ Z_{xi} = F_i \cdot \sin 45^\circ \end{cases}$$

Where there is $i = us, up, ls, \text{ and } lp$. In Fig. 2, $\delta_{us}, \delta_{ls}, \delta_{up},$ and δ_{lp} are upper-starboard, lower-starboard, upper-port, and lower-port rudder angles, respectively. Signs of rudder angles are specified as follows: from the stern, starboard-side rudder-angles are positive and port-side ones are negative.

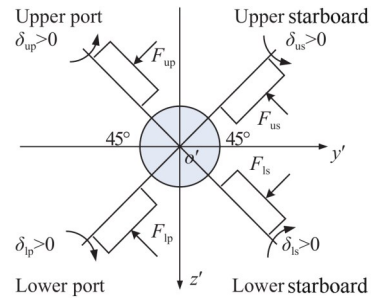


Fig. 2 X-rudder force in the submarine's cross-section

1.2 Spatial steering-motion model of X-rudder submarines

Under the assumption that longitudinal velocity u is constant, spatial steering motion equations of an X-rudder submarine can be established by replacing steering rudder force and stern elevator force in spatial steering motion equations of a C-rudder submarine^[8] with various components $[Y_x, Z_x, K_x, M_x, N_x]$ of X-rudder force. The force and moment of X-rudders are as follows.

$$\begin{cases} Y_x = \frac{1}{2} \rho L^2 u^2 (Y'_{\delta_{us}} \delta_{us} + Y'_{\delta_{up}} \delta_{up} + Y'_{\delta_{ls}} \delta_{ls} + Y'_{\delta_{lp}} \delta_{lp}) \\ Z_x = \frac{1}{2} \rho L^2 u^2 (Z'_{\delta_{us}} \delta_{us} + Z'_{\delta_{up}} \delta_{up} + Z'_{\delta_{ls}} \delta_{ls} + Z'_{\delta_{lp}} \delta_{lp}) \\ K_x = \frac{1}{2} \rho L^3 u^2 (K'_{\delta_{us}} \delta_{us} + K'_{\delta_{up}} \delta_{up} + K'_{\delta_{ls}} \delta_{ls} + K'_{\delta_{lp}} \delta_{lp}) \\ M_x = \frac{1}{2} \rho L^3 u^2 (M'_{\delta_{us}} \delta_{us} + M'_{\delta_{up}} \delta_{up} + M'_{\delta_{ls}} \delta_{ls} + M'_{\delta_{lp}} \delta_{lp}) \\ N_x = \frac{1}{2} \rho L^3 u^2 (N'_{\delta_{us}} \delta_{us} + N'_{\delta_{up}} \delta_{up} + N'_{\delta_{ls}} \delta_{ls} + N'_{\delta_{lp}} \delta_{lp}) \end{cases} \quad (1)$$

where $Y'_{\delta_i}, Z'_{\delta_i}, K'_{\delta_i}, M'_{\delta_i}$, and N'_{δ_i} are dimensionless hydrodynamic coefficients of X-rudder force; ρ is the density of sea water; L is the length of the submarine.

2 Basic structure of steering controller of X-rudder submarine

Many modern aviation and underwater vehicles use over-actuated systems due to the requirements on high reliability. The basic characteristic of such systems is that dimensions of controlling quantities are greater than those of controlled quantities. Controller design of over-actuated systems often adopts a "master controller-control allocator" cascade structure. In other words, a master controller generates to-

tal control force to realize task regulation, while a control allocator maps the total control force as the thrust of each actuator (or deflection of a steering surface) to complete task selection of actuators^[9].

As shown in Fig. 3, for the steering control system of an X-rudder submarine with four independently driven rudder surfaces, dimensions of its controlled quantities are three, including command heading ψ_c , command depth ζ_c , and command trimming θ_c . Steering surfaces include four X-rudder surfaces (for heading control and mainly for trimming control) and one bow elevator surface (mainly for diving-depth control). Therefore, dimensions of controlling quantities are five. Obviously, this system is an over-actuated control system.

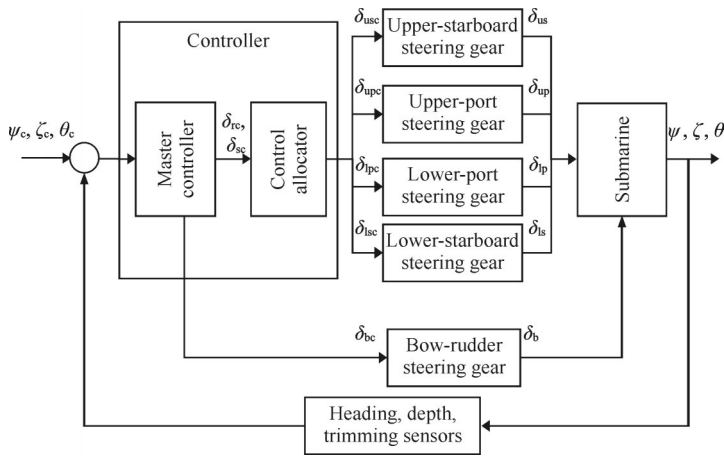


Fig. 3 X-rudder steering control system of submarines

A steering controller consists of a master controller and a control allocator. The master controller generates a command rudder angle δ_{bc} of the bow rudder and a virtual controlling quantity $\mathbf{v} = [\delta_{rc}, \delta_{sc}]^T$ of the stern rudder, $\mathbf{v} \in \mathbf{R}^2$; δ_{rc} and δ_{sc} are angle commands of the equivalent steering rudder and stern elevator, respectively. \mathbf{u} is an actual controlling quantity output from the rudder-angle allocator, namely an X-rudder command angle. Specifically, there is $\mathbf{u} \in \mathbf{R}^k$, where k is the number of dimensions of X-rudder controlling quantities ($k = 2, 3$ and 4), and in the case of $k = 4$, there is $\mathbf{u} = [\delta_{usc}, \delta_{upc}, \delta_{lpc}, \delta_{lsc}]^T$.

The master controller and the control allocator are designed independently. The master controller is only required to generate virtual rudder-angle commands. Therefore, it can employ all kinds of mature control algorithms. The control allocator is required to realize the allocation of control commands under the consideration of physical constraints on steering surfaces. Control allocation is actually a problem of solving constrained linear equations. In other words, it is a problem of solving the actual controlling quali-

ty \mathbf{u} , given the virtual controlling quality \mathbf{v} . In addition, the following conditions are satisfied:

$$\begin{cases} \mathbf{B}\mathbf{u} = \mathbf{v} \\ \underline{\mathbf{u}} \leq \mathbf{u} \leq \bar{\mathbf{u}} \end{cases} \quad (2)$$

Where $\mathbf{B}^{2 \times k}$ is a control-efficiency matrix, with a rank of 2; $\bar{\mathbf{u}}$ and $\underline{\mathbf{u}}$ are the upper and lower limits of control constraints, respectively. For a steering gear system, the above control variables are mainly reflected as rudder-angle and rudder-speed limits.

3 Algorithm of X-rudder control allocation

For convenient discussion, independently driven rudder surfaces of X-rudders are divided into three control configurations: quadruple-rudder (QR), triple-rudder (TR), and double-rudder (DR) control. This totally includes eleven combinations, namely $C_4^4 + C_4^3 + C_4^2 = 11$. This paper selected one typical working condition in TR and DR configurations for design and simulation analysis.

3.1 Analysis of control-efficiency matrix

1) QR: Four rudder surfaces are driven with $k = 4$ and $\mathbf{u} = [\delta_{usc}, \delta_{upc}, \delta_{ipc}, \delta_{isc}]^T$. According to equivalent rudder-angle transition relation, we have

$$\mathbf{B} = \sqrt{2} \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & -0.25 & 0.25 & -0.25 \end{bmatrix}$$

2) TR: Three rudder surfaces are driven with $k = 3$. If we let $\mathbf{u} = [\delta_{usc}, \delta_{ipc}, \delta_{isc}]^T$, we have

$$\mathbf{B} = \sqrt{2} \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & -0.25 \end{bmatrix}$$

3) DR: Two rudder surfaces are driven with $k = 2$. This mode, which belongs to equal-actuated control, can realize heading and trimming control of submarines. Under this configuration of control, there are six specific situations. Here, we consider the case in which only one rudder surface is available in each group of diagonal rudders. If $\mathbf{u} = [\delta_{usc}, \delta_{isc}]^T$, we have

$$\mathbf{B} = \sqrt{2} \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & -0.25 \end{bmatrix}$$

3.2 Algorithm of control allocation

The control-allocation problem described in Eq. (2) can evolve into constrained optimization by introducing a measure criterion^[10]:

$$\min_{\mathbf{u}} \|\mathbf{u}\|_2$$

In this case, the use of two-norm is convenient to take control energy consumption as a cost function. Thus, the control-allocation problem can be transformed into the following quadratic programming problem^[11]:

$$\begin{cases} \min J(\mathbf{u}) = \mathbf{u}^T \mathbf{W}_u \mathbf{u} \\ \mathbf{B}\mathbf{u} = \mathbf{v} \\ \underline{\delta} \leq \mathbf{u} \leq \bar{\delta} \\ \underline{\dot{\delta}} \leq \dot{\mathbf{u}} \leq \bar{\dot{\delta}} \end{cases} \quad (3)$$

where $J(\mathbf{u}) = \mathbf{u}^T \mathbf{W}_u \mathbf{u}$. It is an optimization objective function of matrix two-norm, where \mathbf{W}_u is a weighting matrix of the control quantity \mathbf{u} and $\dot{\delta}$ is rudder speed.

A WLS method was used as the control-allocation algorithm. Essentially, this optimization aims to minimize the control energy consumption of a steering gear on the basis of satisfying control-allocation accuracy. This optimization was described as the following sequential least squares (SLS) problem:

$$\begin{cases} \mathbf{u} = \arg \min_{\mathbf{u} \in \Omega} \|\mathbf{W}_u(\mathbf{u} - \mathbf{u}_d)\|_2 \\ \Omega = \arg \min_{\mathbf{u} \leq \bar{\mathbf{u}}} \|\mathbf{W}_v(\mathbf{B}\mathbf{u} - \mathbf{v})\|_2 \end{cases} \quad (4)$$

where \mathbf{W}_v is a weighted matrix of the virtual control quantity \mathbf{v} ; \arg is a variable; \mathbf{u}_d is a desired controlling quantity, with the same constraint as mentioned above; Ω is a set of allocation solutions with mini-

mum control-allocation errors. The controlling quantity \mathbf{u} with the minimum deflection of a steering surface relative to the desired control quantity \mathbf{u}_d is then solved. By introducing a weight coefficient γ and combining the two equations in Eq. (4), we can transform this problem into a WLS extremum-solving problem, as shown in Eq. (5):

$$\mathbf{u} = \arg \min_{\mathbf{u} \leq \bar{\mathbf{u}}} \left(\|\mathbf{W}_u(\mathbf{u} - \mathbf{u}_d)\|_2^2 + \gamma \|\mathbf{W}_v(\mathbf{B}\mathbf{u} - \mathbf{v})\|_2^2 \right) \quad (5)$$

The advantage of a WLS method is that two-step operations of a SLS method are simplified into one step, reducing computing time. Thus, an active-set algorithm can be used to solve this WLS problem with constraints^[12-13].

4 Verification through simulation

In order to verify the validity of the control-allocation algorithm, this paper simulated diving-depth and heading control by using the submarine parameters mentioned in Reference [14], based on the spatial model of steering motion of an X-rudder submarine. Specifically, derivatives of X-rudder angles were calculated from those of C-rudder angles through the equivalent rudder-angle method. The master controller adopted an improved sliding-mode control algorithm based on a nonlinear disturbance observer (NDO)^[15].

Parameters of the control allocator were as follows: $\mathbf{W}_v = \mathbf{I}^{2 \times 2}$, $\mathbf{W}_u = \mathbf{I}^{k \times k}$, $\mathbf{u}_d = \mathbf{0}^{k \times 1}$, and $\gamma = 10^6$, where \mathbf{I} is a unit matrix and $\mathbf{0}$ is a zero matrix. Control constraints are as follows: rudder-angle limitation of $-30^\circ \leq \delta_i \leq 30^\circ$ and rudder-speed limitation of $-3 (^\circ)/s \leq \dot{\delta}_i \leq 3 (^\circ)/s$.

4.1 Simulation of turning at certain depth

Settings for simulating turning of a submarine at certain depth were as follows: speed of 10 kn, initial heading of $\psi_0 = 0^\circ$, and command heading of $\psi_c = 20^\circ$. In addition, the submarine was in a balanced state. Based on the simulation of turning at a certain depth under three configurations of available rudder surfaces, curves of rudder-angle δ allocation, heading ψ response, and heeling φ response were obtained as shown in Figs. 4-6. Table 1 lists the main performance parameters of heading and heeling.

According to the simulation results, although steering with diagonally coordinated operation of rudder surfaces is adopted under both QR and TR configurations, rudder angles in the case of QR are smooth without oscillation, while those in the case of TR oscillate slightly. By contrast, rudder angles oscillate

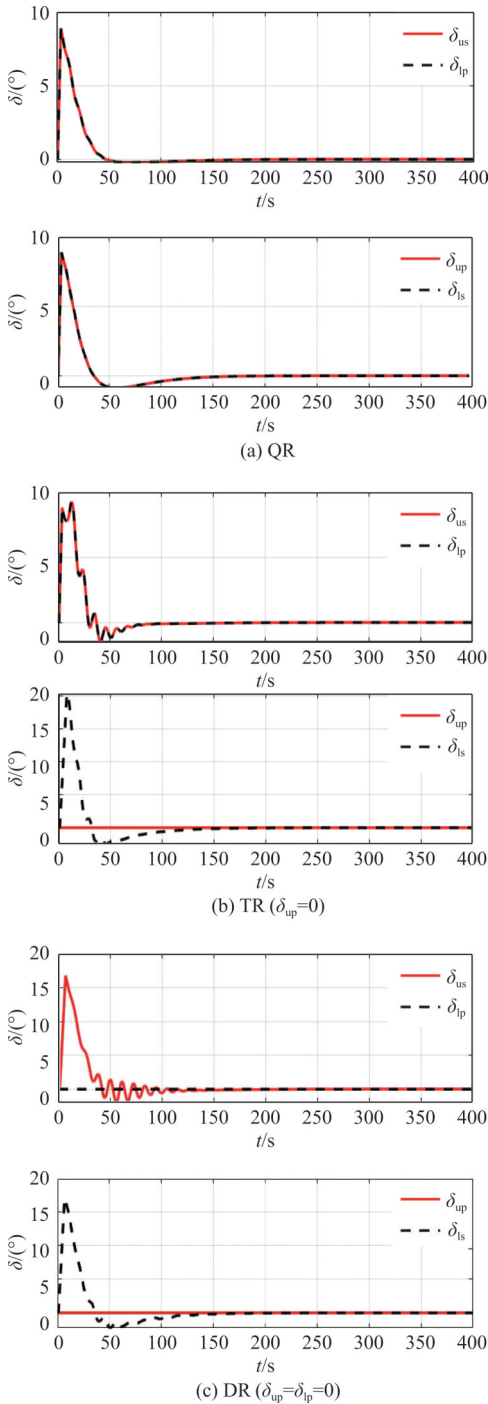


Fig. 4 Rudder angle allocation on turning at certain depths

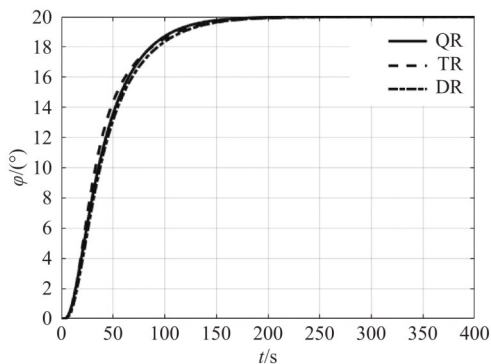


Fig. 5 Heading response on turning at certain depths

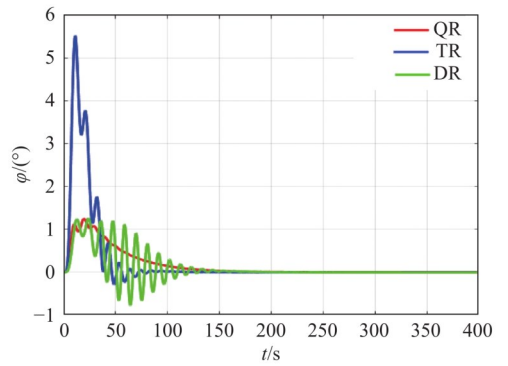


Fig. 6 Heeling angle response on turning at certain depths

Table 1 Control effect comparison on turning at certain depths under three kinds of available rudder configuration

Parameter	Rudder-surface configuration		
	QR	TR	DR
Heading overshoot / (°)	0	0	0
Heading adjustment time/s	107.5	114.5	128.7
Peak heeling angle / (°)	1	5.8	1.8
Number of heeling oscillations	3	8	16

obviously in the case of DR (Fig. 4). In the case of turning under three configurations of available rudder surfaces, the submarine turns smoothly (Fig. 5 and Table 1). The heeling angle in the case of QR oscillates slightly, and that in the case of TR has the greatest peak, with insignificant oscillation. By contrast, the heeling angle in the case of DR oscillates obviously (Fig. 6 and Table 1). Thus, with the decrease in steering surfaces, maneuverability of the submarine gradually degrades, and both heeling oscillation and rudder-angle oscillating tend to be serious.

4.2 Simulation of diving

Settings for simulating diving motion of a submarine were as follows: speed of 10 kn, initial depth of $\zeta_0 = 50$ m, command depth of $\zeta_c = 100$ m, and command trimming of $\theta_c = 0^\circ$. In addition, the submarine was in a balanced state. Based on the simulation of directional diving under three configurations of available rudder surfaces, results of rudder-angle δ allocation, depth ζ response, trimming θ response, and heeling φ response were obtained as shown in Figs. 7–10.

According to the simulation results, although steering with diagonally coordinated operation of rudder surfaces is adopted under both QR and TR configurations (Fig. 7), no heeling is observed in the case of QR, while heeling with smooth variation is observed

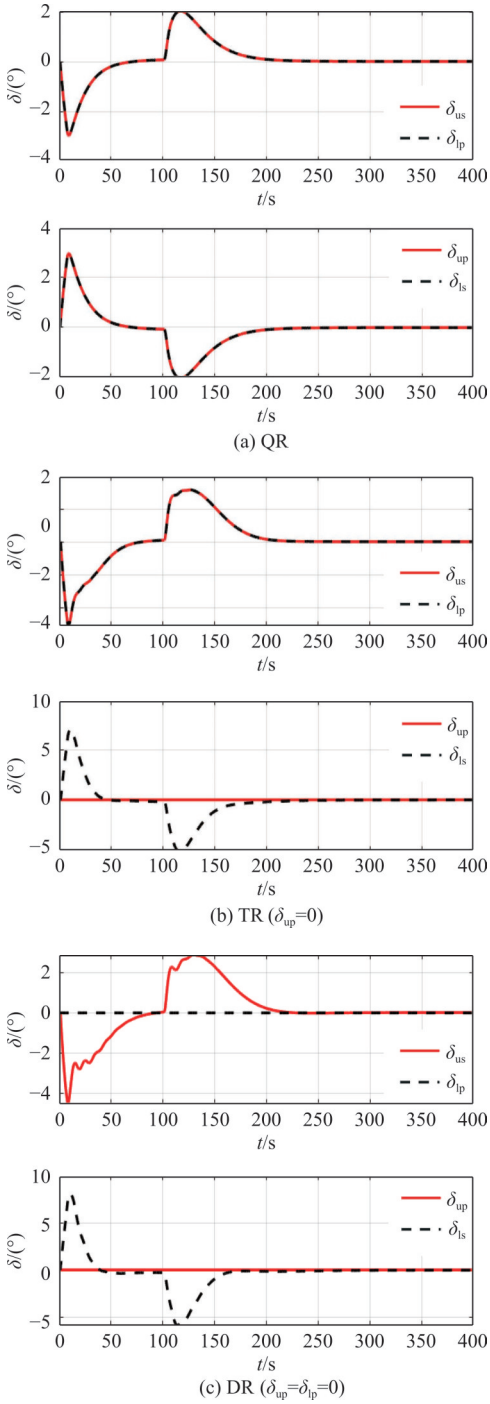


Fig. 7 Rudder angle allocation on diving

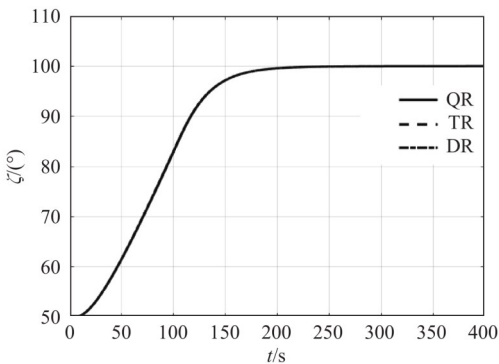


Fig. 8 Depth response on diving

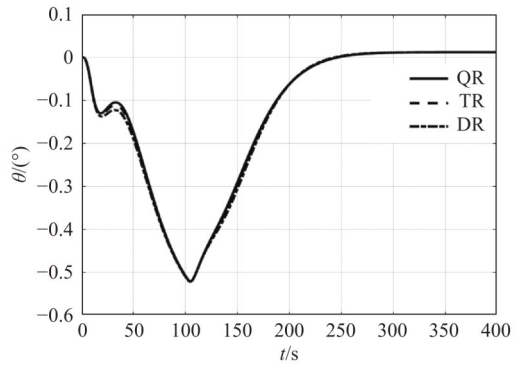


Fig. 9 Pitch angle response on diving

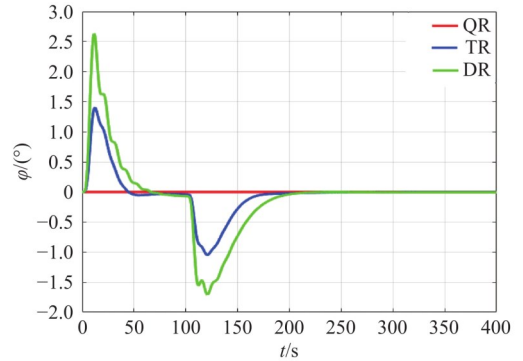


Fig. 10 Roll response on diving

in the case of TR. Thus, steering under both configurations is smooth. By contrast, the heeling peak of DR is the greatest, and both heeling oscillation and rudder-angle oscillating are observed (Figs. 7 and 10). There is little difference in the performance of depth and trimming control under the three configurations of available rudder surfaces (Figs. 8 and 9). This is because the bow rudder plays an important role in depth control. However, X-rudder control laws in the case of depth control with a single stern rudder need to be further studied.

It should be noted that TR and DR control have different steering gear configurations, and the above simulation curves were obtained under given steering gear configurations. Under different steering gear configurations, control-allocation algorithms are the same, but performance of heading and depth control will be different.

5 Conclusions

In view of X-rudder steering control of a submarine, this paper adopted a "master controller-control allocator" cascade control structure. Specifically, the master controller used was an existing improved sliding-mode controller based on NDO, and the control allocator employed a control-allocation algorithm through WLS. This algorithm is applicable to various

steering gear configurations. According to the simulation results, under a given steering gear configuration, with the decrease in steering surfaces, diving performance of the submarine has no difference. However, steering performance reduces slightly, and both heeling oscillation and rudder-angle jittering tend to be serious. In fact, X-rudders have better heeling-control abilities, which can be realized by designing control laws of differential rudder angles. In this way, maneuverability of a submarine in the case of fewer available rudder surfaces and strong motility can be improved. Further research will be carried out in this regard.

References:

- [1] HU K, WANG S Z. Design research of the manipulative parameter of the X ruder for underwater vehicle [J]. Ship & Ocean Engineering, 2008, 37 (3): 127-131 (in Chinese).
- [2] ZENG J B. Research and application of the control system for a portable autonomous underwater vehicle [J]. Robot, 2016, 38 (1): 91-97 (in Chinese).
- [3] ZHANG Y, ZENG J, LI Y, et al. Research on reconstructive fault-tolerant control of an X-rudder AUV [C]. // Conference on OCEANS 2016. Monterey, CA: MTS/IEE, 2016.
- [4] ZHANG Y H, LI Y M, SUN Y S. Design and simulation of X-rudder AUV's motion control [J]. Ocean Engineering, 2017, 137: 204-214.
- [5] ZHANG Y, LI Y, ZHANG G, et al. Design of X-rudder autonomous underwater vehicle's quadruple-rudder allocation with Lévy flight character [J]. International Journal of Advanced Robotic Systems, 2017, 14 (6): 1-15.
- [6] The Editorial Office of Shipbuilding of China. Dictionary of ship dynamics [M]. Shanghai: The Editorial Office of Shipbuilding of China, 1981 (in Chinese).
- [7] SNAME. Nomenclature for treating the motion of a submerged body through a fluid [R]. SNAME Technical and Research Bulletin 1-5, 1952.
- [8] GERTIER M, HAGEN R, GERTLER M. Standard equations of motion for submarine simulation: DTMB2510 [R]. Washington DC: David Taylor Research Center, 1967.
- [9] HARKEGARD O. Backstepping and control allocation with applications to flight control [D]. Linköpings: Linköpings Universitet, 2003.
- [10] BUFFINGTON J M. Tailless aircraft control allocation [C]. //AIAA Guidance, Navigation, and Control Conference. New Orleans, LA: AIAA 1997: 737-747.
- [11] ENNS D. Control allocation approaches [C]. //Proceedings of AIAA Guidance Navigation and Control. Boston, MA: AIAA, 1998: 98-108.
- [12] HARKEGARD O. Efficient active set algorithms for solving constrained least squares problems in aircraft control allocation [C]. //Proceedings of the 41st IEEE Conference on Decision and Control. Las Vegas, NV: IEEE, 2002.
- [13] XIONG L, YU Z P, JIANG W, et al. Research on vehicle stability control of 4WD electric vehicle based on longitudinal force control allocation [J]. Journal of Tongji University (Natural Science), 2010, 38 (3): 417-421 (in Chinese).
- [14] BABA OGLU O K. Design an automatic control system for a submarine [D]. Monterey, CA: Naval Postgraduate School, 1988.
- [15] LU B J. Research on improved sliding mode control algorithm and its application in submarine manoeuvring [D]. Wuhan: Naval University of Engineering (in Chinese).

潜艇 X 舵控制分配器设计和仿真

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摘要: [目的] 为解决 X 舵潜艇在不同可用舵面条件下的操舵控制问题, 开展 X 舵潜艇的操舵控制分配器设计。[方法] 操舵控制系统采用“主控制器—控制分配器”级联控制结构, 主控制器采用已有的成熟算法, 将舵角控制分配问题转换为受舵角限幅、舵速限幅约束的加权最小二乘法 (WLS) 问题进行求解, 设计受限控制分配算法, 并分别在四舵、三舵、双舵条件下进行定深旋回、潜浮运动仿真, 以验证控制分配算法的正确性。[结果] 仿真结果表明, 在给定的舵机配置条件下, 随着操舵面的减少, 潜浮性能并无差异, 转向性能略有降低, 横倾振荡及舵角抖动趋于严重。[结论] 所提控制分配算法施舵合理, 适用于各种舵机配置情况。

关键词: 潜艇; X 舵; 控制分配; 加权最小二乘法